$\underline{Q}$: $\frac{d}{dx}\left[e^{5x}\right]$

$$h(x) = e^{5x} = f(g(x))$$

where $f(u) = e^{u}$ $\quad g(x) = \underline{5x = u}$

The Chain Rule

If g is differentiable at x and f is differentiable at $g(x)$, then the composition $F(x) = f(g(x))$ is differentiable at x and

$$\boxed{F'(x) = f'(g(x)) \cdot g'(x)}$$

If $\underline{u = g(x)}$ and $y = f(u)$, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

replace u's afterward

Ex: ① $h(x) = e^{5x}$

$y = f(u) = e^u$

$u = g(x) = 5x$

$$h'(x) = \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = e^u \cdot 5 = 5e^{5x}$$

$$\frac{d}{dx}\left[e^{5x}\right] = 5e^{5x}$$

② $h(x) = \sin(x^2+4)$

$u = g(x) = \underline{x^2+4}$

$y = f(u) = \sin(u)$

$$h'(x) = f'(u) \cdot g'(x)$$

$$= \cos(u) \cdot (2x)$$

$$= 2x\cos(x^2+4)$$

③ $F(x) = \sqrt{x^3+4x}$

$u = g(x) = x^3+4x$

$y = f(u) = \sqrt{u} = u^{1/2}$

$$F'(x) = \frac{1}{2}u^{-1/2} \cdot (3x^2+4)$$

$$= \frac{1}{2}(x^3+4x)^{-1/2}(3x^2+4) = \frac{3x^2+4}{2\sqrt{x^3+4x}}$$

④ $f(x) = \underline{2^x} = e^{\ln(2^x)} = \underline{e^{x\ln(2)}}$ ← just a #

$u = x\ln(2)$

$y = e^u$

$f'(x) = e^u \cdot \ln 2$

$= \underbrace{e^{x\ln 2}} \cdot \ln 2$

$= 2^x \cdot \ln 2$

$\frac{d}{dx}[2^x] = 2^x \cdot \ln 2$

$$\boxed{\frac{d}{dx}[b^x] = b^x \cdot \ln b}$$

⑤ $f(x) = \underbrace{(2x-3)^4}\underbrace{(x^2+x+1)^5}$

$f'(x) = \frac{d}{dx}[(2x-3)^4](x^2+x+1)^5 + (2x-3)^4 \cdot \frac{d}{dx}[(x^2+x+1)^5]$

$= (4(2x-3)^3 \cdot 2)(x^2+x+1)^5 + (2x-3)^4(5(x^2+x+1)^4 \cdot (2x+1))$

$= \boxed{8(2x-3)^3(x^2+x+1)^5 + 5(2x-3)^4(x^2+x+1)^4(2x+1)}$

(6) $y = \sin\left(e^{x^2}\right)$

$v = h(x) = x^2$

$u = g(v) = e^v$

$y = f(u) = \sin(u)$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx}$$

$$= \cos(u) \cdot e^v \cdot 2x = \cos(e^v) \cdot e^v \cdot 2x$$

$$= \cos\left(e^{x^2}\right) \cdot e^{x^2} \cdot 2x$$

$$\frac{d}{dx}\left[\sin\left(e^{x^2}\right)\right] = \cos\left(e^{x^2}\right) \cdot \frac{d}{dx}\left[e^{x^2}\right]$$

$$= \cos\left(e^{x^2}\right) \cdot e^{x^2} \cdot \frac{d}{dx}\left[x^2\right]$$

$$= \cos\left(e^{x^2}\right) \cdot e^{x^2} \cdot 2x$$